## Raveling Tic-Tac-Toe

Some games may support the learning of mathematics in one way or another. It is important, however, to identify the educational component in the game as playing a game, by itself, may not lead to further learning. Here, an analysis of the mathematical potential in Tic-tac-toe is analyzed through the lens of the RaPID model.

Many people are familiar with tic-tac-toe, which is a popular game among children at elementary school level. The game has the potential to involve some reasoning skills, including deductive logic and systematic analysis. However, kids may not reach this potential just by playing the game.

## The RaPID model

RaPID stands from Raveling, Prompting, Interpreting and Deciding. Figure 1 and Figure 2 are included as a quick reference to the model. Figure 1 summarizes the principles and practices comprised by the model and Figure 2 provides examples of the implications of using the model in terms of things to do and things to avoid. This document will focus only on the mathematical structure through unraveling the mathematical potential in Tic-Tac-Toe.

| Principles | Practices |
|---|---|
| P1: Mathematical Structure | Raveling: Long Term (R-a) & Short-Term (R-b) |
| P2: Mindset | Prompting: Change (P-a) & Choice (P-b) |
| P3: Working Memory | Interpreting: Focused (I-a), Attentive to All (I-b) |
| P4: Change | Deciding: Responsive (D-a) & Unifying (D-b) |
| P5: Overlap | |

Figure 1. Math Minds Principles

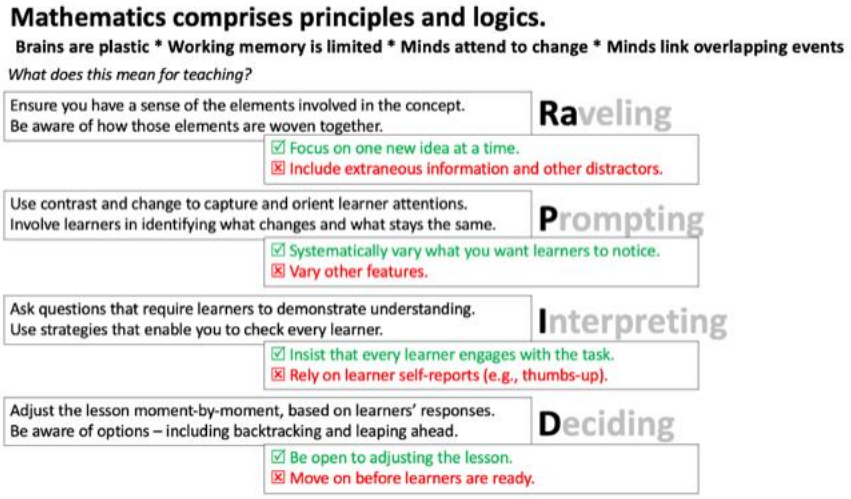


Figure 2. Examples of what to do and what to avoid in the RaPID model

## Analysis

The RaPID model emphasizes the identification of critical features that students have to discern in order to learn something (we call these critical discernments). Let's unravel the discernments involved in playing Tic-tac-toe successfully: that is, to either win or avoid losing. Table 1 summarizes some categories of playing the game. Each category involves some discernments that lead to decisions for a successful play – that is either win or avoid losing.

**Table 1. Categories of playing Tic-tac-toe**

| Categories of playing | Discernments and actions involved |
|---|---|
| (1) Random | Understanding the rules of the game |
| (2) Simple attack | A player can place two marks in a row, column or diagonal in a deliberate attempt to win. |
| (3) Defense | A player can prevent the opponent to win bay marking the position that would be the third opponents' mark for a win. |
| (4) Forced move | A player can force the next opponent's move (to avoid losing).<br>A player can try to avoid the opponent to create a forced move. |
| (5) Double attack | Double attack is a configuration in which the next player loses: It involves two forced moves simultaneously. |
| (6) Identify success strategies through playing | This involves the identification of success configurations, as well as the deliberate attempts to use them when playing. |
| (7) Identify success strategies through analyzing configurations | This involves an analysis of different configurations in an attempt to find success strategies. |
| (8) Identify success strategies through a systematic analysis of configurations | This involves a systematic analysis of all possible configurations. |

When learning to play the game, someone may play randomly. This involves understanding the rules of the game: winning or losing is a matter of chance. However, players usually notice quickly that in order to win, there should first be two marks in a row, column or diagonal, as in Figure . This has two implications for playing: first, players can deliberately place two marks in such positions attempting to win (*simple attack*); second, players can prevent the opponent's victory by marking the space that would be used for winning (*defense*).

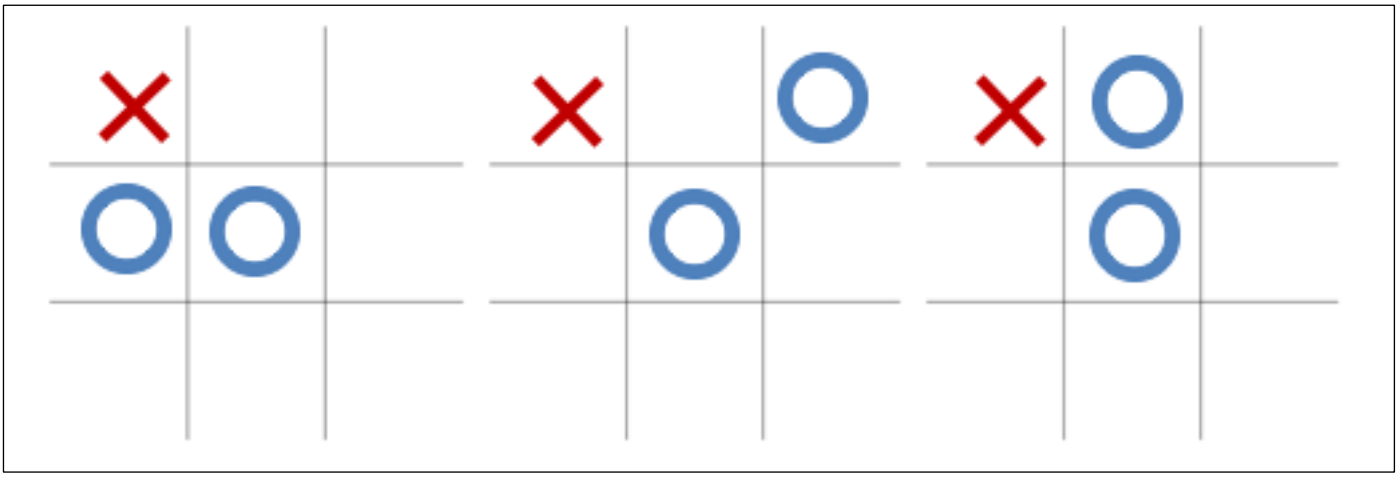

Figure 1. Two marks (circles) in a row, column or diagonal

Once understanding the simple attack and the defense, players notice that certain configurations, such as in Figure 1, force the next player to choose one specific position in the next move to avoid losing. This corresponds to the *forced move* category in Table 1. A master player not only identifies these configurations, but also anticipates the moves to create a forced move, or to prevent the opponent from creating one. A double forced move, or *double attack*, leaves the opponent in a losing situation regardless of the next move: Figure  shows examples of double attack configurations in which no matter where the next player places a mark (red x),  the player loses.

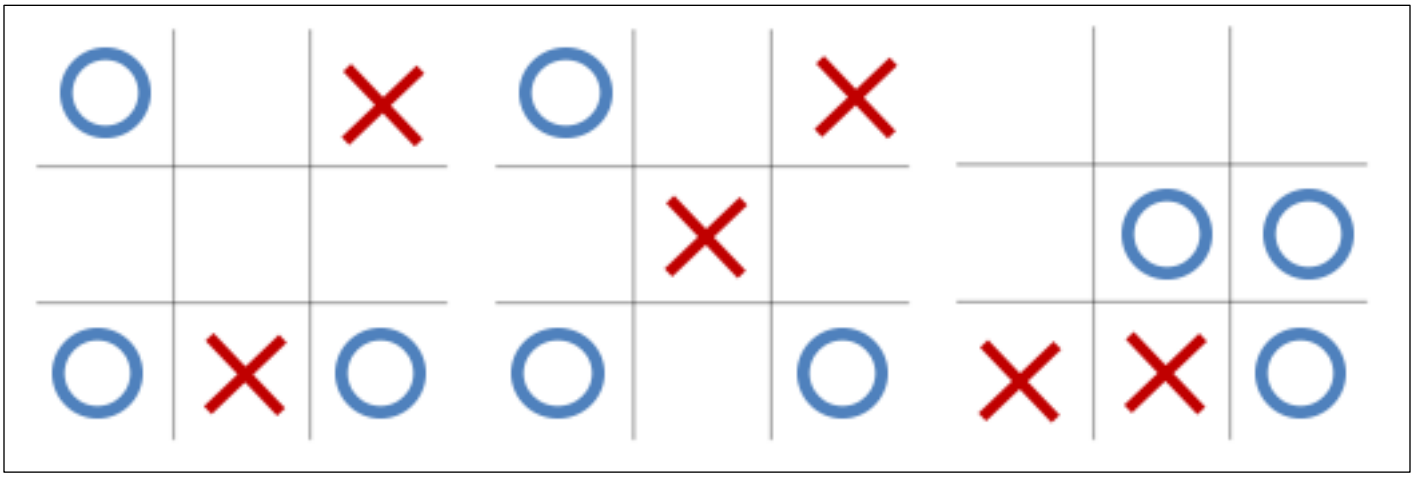

Figure 2. Examples of double attack configurations

Anticipating in this case leads to *identifying learning strategies*, which can be done through *playing multiple times* and *analyzing configurations*. While many people would do the former, fewer people engage in the later. At this stage, deductive logic starts to play an important role.

Playing multiple times allows variation in the configurations that players can consider in identifying success strategies. This process can involve noticing patterns (e.g. choosing the centre has higher chances of success) and apply deductive logic (anticipating certain configurations). However, in this approach the learner may not be exposed to all possible configurations or may not be able to compare them.

There is great learning potential in a systematic analysis of possible configurations. Analyzing configurations may or may not involve playing. One person may study configurations from a played game and study implications of certain moves through analyzing different configurations without playing with another person.

All possible configurations can be analyzed in a *systematic* way, which is the last category in Table 1. There are different ways of doing this. However, variation in any case should be deliberate, so we can be sure that all cases have been exhausted. As this is not usually done independently by learners, the teacher, or the adult, can prompt the learners to conduct a systematic analysis in order to identify the winning strategies.